

# Multiphase flow in porous media

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**Multiphase flow** concerns the flow of two or more immiscible fluid phases in a porous medium. Therefore several fluids (e.g., water, air, mineral oil, organic solvents etc.) are present. In unconfined aquifers a capillary zone is present above the groundwater table (Figure 1), in which both water and air phase may be present at the same time. The resulting flow is a **two-phase flow**. If air flow can be neglected the resulting flow is called **unsaturated flow**. This chapter concentrates on the basic principles of multiphase flow.

## 1. Flow of water and air in the capillary zone

In an **unconfined aquifer** the upper boundary is the groundwater table. The position of the groundwater table is usually defined as the location with atmospheric pressure (zero relative pressure). The domain above the groundwater table is the **capillary zone**. In principle the capillary zone is hydraulically unsaturated by the simultaneous presence of water and air in a control volume. However, part of the capillary zone can still be hydraulically saturated (saturated capillary fringe) (Fig. 1). In saturated groundwater models for flow in unconfined aquifers the two domains are uncoupled. The coupling is performed with the help of the storativity concept and the areal recharge rate.

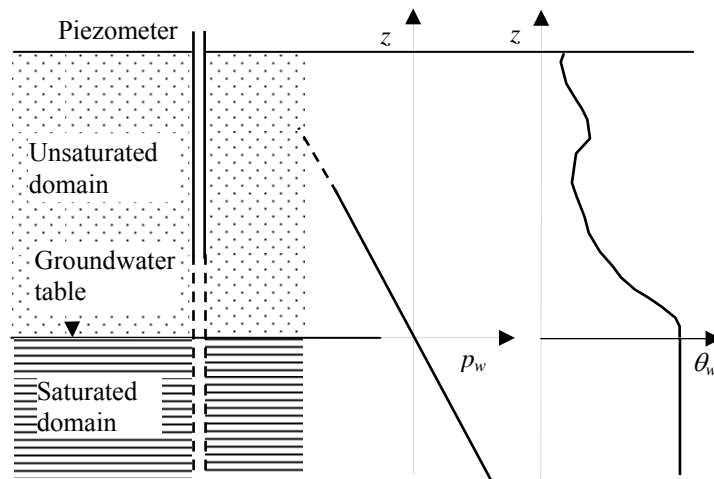


Fig. 1: Capillary zone above a groundwater table; vertical water pressure profile  $p_w(z)$  and vertical profile of the volumetric water content  $\theta_w(z)$  (schematic).

### 1.1 Flux equations for flow in the capillary zone

Hydraulically unsaturated porous media are characterized by the presence of a continuous air phase besides a continuous water phase. In both fluid phases a flow can take place. The

spatial distribution of both phases strongly depends on the wetting properties with respect to the solid phase (e.g., the grains). The volumetric fraction of both phases can be time dependent. The volumetric fraction of the water phase in a control volume is called **volumetric water content**  $\theta_w$  (Water volume per unit volume of porous medium). In the unsaturated case  $\theta_w \leq n$ , where  $n$  is the porosity. Accordingly,  $\theta_a$  is the volumetric air content.

For simplicity, the **flux equations** for the water and the air phases can be adopted from saturated flow conditions by analogy and adjusted accordingly. Consider the following situation. A vertical column with a homogeneous porous medium is recharged by a uniform steady-state infiltration rate at the upper inflow face. If the column is long enough a practically uniform flow is established with constant water content, provided the infiltration rate is smaller than the hydraulic conductivity of the porous medium at saturation. Such a water flow system is characterized by two kinds of boundaries within the microscopic flow system. On the one hand the solid phase represents the boundary as in saturated flow. On the other hand the air-water interface is a boundary. The difference to saturated conditions lies in the fact that the volumetric water content is smaller than porosity. Furthermore, the resistance at the water-air interface is different. Nevertheless it can be assumed that **Darcy's law for the water phase** (index  $w$ ) can be generalized for constant water density:

$$\mathbf{v}_w = -\mathbf{K}_w(S_w) \nabla \left( z + \frac{p_w}{\rho_w g} \right)$$

The parameter  $\mathbf{v}_w$  is the specific water flux,  $\mathbf{K}_w$  is the hydraulic conductivity tensor,  $z$  is the vertical coordinate,  $p_w$  is the water pressure,  $\rho_w$  the water density and  $g$  the acceleration constant. Similarly, **Darcy's law for the air phase** (index  $a$ ) can be postulated as follows:

$$\mathbf{v}_a = -\mathbf{K}_a(S_a) \nabla \left( z + \frac{p_a}{\rho_a g} \right)$$

The parameter  $\mathbf{v}_a$  is the specific air flux,  $\mathbf{K}_a$  is the air conductivity tensor.  $S_{wa}$  is the water saturation and  $S_a$  the air saturation, with:

$$S_w = \frac{\theta_w}{n} \quad S_a = \frac{\theta_a}{n}$$

Both saturations are fractions of the pore space and therefore have to fulfil the condition  $S_w + S_a = 1$ .

It can be expected that the hydraulic conductivity  $\mathbf{K}$  strongly depends on the water or air content  $\mathbf{K}_w(S_w)$  or  $\mathbf{K}_a(S_a)$ . The formulation above does not take into account the state of the other phase within the porous medium.

In a further step **non-uniform and transient phase distribution** is adopted conceptually. Consequently the equations above are accepted as approximations for expressing the flux of fluids in the capillary zone. However, it can be expected that in the transient case the hydraulic conductivity  $K$  additionally depends also on the temporal change of the water or air content, e.g.,  $K_w(S_w, \partial S_w / \partial t)$  (Stauffer, 1998).

Similarly the **variable fluid density** can be taken into account in the **generalized Darcy law** as approximation:

$$\mathbf{v}_w = \frac{\mathbf{k}_w(S_w)}{\mu_w} [-\nabla p_w - \rho_w \mathbf{g}]$$

$$\mathbf{v}_a = \frac{\mathbf{k}_a(S_a)}{\mu_a} [-\nabla p_a - \rho_a \mathbf{g}]$$

The parameter  $\mathbf{k}$  is the permeability tensor, which depends on the water and air content. The parameter  $\mu_w$  and  $\mu_a$  are the dynamic viscosities for water and air, and  $\mathbf{g}$  is the acceleration vector  $\mathbf{g}=(0, 0, g)$ . Moreover, it has to be noted that the air permeability can depend on the air pressure. This is particularly true for fine-textured soils (so called Klinkenberg effect, Corey, 1977). Consequently the permeability for water and air can differ, as  $\mathbf{k}_w(S_w)$  and  $\mathbf{k}_a(S_a)$ .

## 1.2 Mass balance equation for the capillary zone

The **mass balance for the water and the air phase** can be formulated as follows:

$$\nabla \cdot (\rho_w \mathbf{v}_w) = -\frac{\partial (nS_w \rho_w)}{\partial t}$$

$$\nabla \cdot (\rho_a \mathbf{v}_a) = -\frac{\partial (nS_a \rho_a)}{\partial t}$$

where  $\nabla \cdot$  is the divergence operator and  $t$  is the time. The above formulation does not yet taken into account phase exchange processes between the phases, due to, e.g., evaporation or condensation of water, or air dissolution in water. Both equations are again coupled by the requirement that  $S_w + S_a = 1$  (see above).

If **constant fluid density and locally constant porosity** can be assumed, the mass balance equations reduce to **volume balance equations** for the water and the air phase:

$$\nabla \cdot \mathbf{v}_w = -n \frac{\partial S_w}{\partial t}$$

$$\nabla \cdot \mathbf{v}_a = -n \frac{\partial S_a}{\partial t}$$

By inserting Darcy's law into the volume balance equations, **non-linear differential equations of second order** are obtained:

$$\nabla \cdot \left( \mathbf{K}_w(S_w) \nabla \left( z + \frac{p_w}{\rho_w g} \right) \right) = n \frac{\partial S_w}{\partial t}$$

$$\nabla \cdot \left( \mathbf{K}_a(S_a) \nabla \left( z + \frac{p_a}{\rho_a g} \right) \right) = n \frac{\partial S_a}{\partial t}$$
(1)

If furthermore the influence of the air phase on water flow is neglected, the second equation reduces to  $p_a=0$ . The first equation describes flow of water in an uncoupled manner. It is known as **Richards equation**:

$$\nabla \cdot \left( \mathbf{K}_w(S_w) \nabla \left( z + \frac{p_w}{\rho_w g} \right) \right) = n \frac{\partial S_w}{\partial t}$$

Equation system 1 contains the four **variables**  $S_w$ ,  $S_a$ ,  $p_w$  and  $p_a$ . Besides the condition  $S_w + S_a = 1$  a further relation is required. This further relationship is the **relation between water content or water saturation and the water pressure  $p_w$  and air pressure  $p_a$** . The relation is known as water retention curve  $S_w(p_w, p_a)$ .

### 1.3 The water retention curve

Consider first the **hydrostatic state** within an unsaturated vertical column of a porous medium. The state is characterized by a lineal vertical water pressure distribution:

$$p_w = p_{w0} - \rho g z$$

The variable  $p_{w0}$  is the water pressure at a reference location with vertical coordinate  $z_0$  with zero water pressure. Air pressure is zero everywhere in the sample with  $p_a = 0$ . The water saturation at location  $z$  is  $S_w(z)$ . It is assumed in this case that the water phase is the **wetting phase** toward the solid phase. Therefore air is the non-wetting phase. Within the pores the interface between water and air is curve due to **interfacial tension**. The resulting local **radius of curvature** can be described by mechanical equilibrium (Corey, 1977) and depends on the pressure difference (pressure step at the interface):

$$p_c = p_a - p_w$$

The variable  $p_c$  is called **capillary pressure**. Note that in the considered hydrostatic situation water pressure is negative and air pressure is zero. Therefore the capillary pressure  $p_c$  is positive. The condition of mechanical equilibrium leads to the **capillary pressure-pore radius concept** for a capillary with radius  $r$ :

$$p_c = \frac{2\sigma_{w,a}}{r} \cos \alpha \quad (2)$$

The parameter  $\sigma_{w,a}$  is the interfacial tension between water and air, and  $\alpha$  is the wetting angle. The concept is often applied as approximation for pores with an inner radius  $r$ . Since the local curvature of the interface depends on the geometrical configuration of the pore space the capillary pressure depends on the water saturation  $S_w(p_c)$ . With increasing capillary pressure the radius of the interface is decreasing and the pore is draining. Therefore water saturation is decreasing. The opposite is true for a decrease of the capillary pressure. The relationship  $S_w(p_c)$  has to be determined **experimentally** (see e.g., Corey, 1977). However, the relation is subject to **hysteresis effects**. This means that the relation for, e.g., a drainage process differs for a wetting process.

It is usually assumed in models that the relation  $S_w(p_c)$  is **identical for static, steady-state and transient conditions**. Furthermore, hysteresis effects are often neglected. Therefore it is assumed that the relation is unique. However, it was shown this is not always the case for

transient conditions (Stauffer, 1998). Therefore dynamic effects can be present thus leading to a form  $S_w(p_c, \partial S_w/\partial t)$ .

In order to parameterize the relation  $S_w(p_w)$  and to use it in flow models there is a need for a mathematical representation of the relation. A frequently used model, which is partially based on theoretical considerations, is the approach of **Brooks and Corey** (1966):

$$S_{w,e} = \frac{S_w - S_{w,r}}{1 - S_{w,r}} = \left( \frac{p_b}{p_c} \right)^\lambda \quad \text{for } p_c \geq p_b$$

$$S_{w,e} = 1 \quad \text{for } 0 \leq p_c \leq p_b$$

The parameter  $S_{w,r}$  is residual water saturation. It is related to the water content, which cannot be further reduced by increasing capillary pressure. The remaining water is therefore immobile. The **effective saturation**  $S_{w,e}$  is related to the water, which is mobile. The parameter  $p_b$  can be interpreted as capillary pressure at which an originally saturated porous medium is starting to drain (air entry pressure). It therefore corresponds to the largest interconnected pore size. The exponent  $\lambda$  is often considered as a measure for the pore size distribution. Large values of  $\lambda$  typically correspond to a narrow distribution and vice versa. The relation of Brooks and Corey is well suited to adequately model the **concave part of the water retention curve** during drainage processes in sand packings or sandy soils.

However, the relation  $S_w(p_c)$  may show **convex-concave shape**, mainly during wetting processes. In order to represent this behaviour the relation of Brooks und Corey can be extended (Stauffer und Dracos, 1984) to:

$$p_c = p_b \left( \frac{S_w - S_{w,r}}{1 - S_{w,r}} \right)^{-\frac{1}{\lambda}} \left( \frac{S_{w,m} - S_{w,w}}{S_{w,m} - S_{w,r}} \right)^\delta \quad \text{for } p_c \geq 0$$

The parameter  $\delta$  can be interpreted as responsible for the distribution of the pore classes with discontinuous air phase, whereas  $\lambda$  describes the pores with continuous air phase. The parameter  $S_{w,m}$  is the maximum saturation and corresponds to the fully saturated or quasi-saturated state with  $S_w < 1$  with insular air (entrapped air bubbles). For  $\delta=0$  the original Brooks and Corey relation is obtained.

In natural soils the approach of **van Genuchten** (1980) is often used for the representation of convex-concave retention curves:

$$S_{w,e} = \left( \frac{1}{1 + \left( \alpha \frac{p_c}{\rho g} \right)^n} \right)^m \quad \text{for } p_c \geq 0$$

with the parameters  $\alpha$ ,  $n$  und  $m$ . Usually  $m$  is taken as  $m=1-1/n$ .

### 1.3.1 Hysteresis in the water retention curve

The hysteresis phenomenon in the retention curve  $S_w(p_c)$  implies that the relation is not unique but depends on the history of the preceding wetting and drainage processes. A relatively simple cycle is described in Figure 2 for a sand packing. A saturated porous medium (with water saturation  $S_w=1$ ) is drained down to the residual saturation  $S_{w,r}$  (first boundary drainage curve). It follows a wetting process (boundary wetting or imbibition curve) until zero capillary pressure is obtained ( $p_c=0$ ). The corresponding water saturation  $S_w(p_c=0)=S_{w,m}$  is smaller than one (quasi saturation with entrapped air bubbles). It follows a new drainage process, down to the residual saturation (second boundary drainage curve). The three curves are not identical.

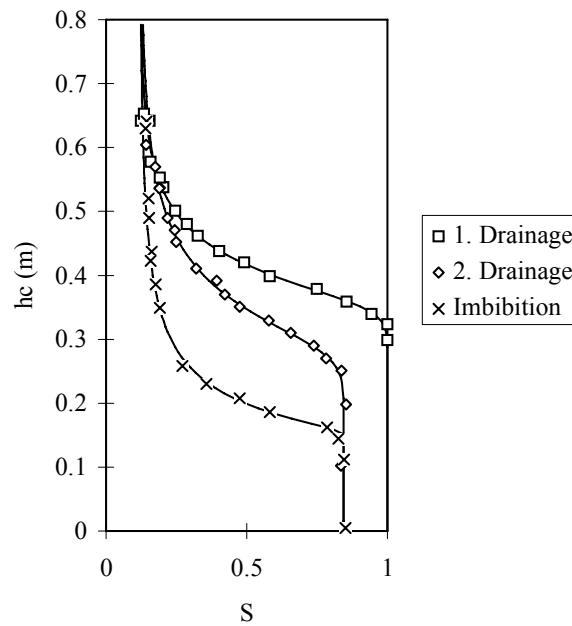


Fig. 2 Boundary drying and wetting (imbibition) curves of the water retention curve for a sand packing; capillary pressure head  $h_c=p_c/(\rho_w g)$  as a function of the water saturation  $S_w$  (Stauffer, 1997).

The effect of the previous history of the process can even be more complicated. Wetting curves after a partial drainage of an initially saturated porous medium are **primary wetting curves**. Drainage curves after a partial wetting process are **primary drying curves**. Starting process on these primary curves lead to secondary, tertiary etc. curves. The **general case** is given by a **series of alternating drying and wetting processes**.

Hysteresis can be attributed to the following effects:

- A single pore can exhibit the same capillary pressure for the fully saturated and the empty pore. Consequently, for many pores, the saturation can be different for the same capillary pressure.
- During wetting of large pores air bubbles can be entrapped. (insular air).
- During drainage processes single pores may remain saturated.
- During drainage and wetting a different wetting angle may be effective.

## 1.4 Hydraulic conductivity characteristics

As already stated before the hydraulic conductivity  $K(S)$  depends on the water saturation  $S$ . Based on Kozeny's concept **Brooks and Corey** (1966) postulated the following relation of  $K_w(S_w)$  for drainage processes:

$$K_w(S) = K_w(S_w = 1) S_e^\varepsilon \quad \text{with } \varepsilon = 3 + 2/\lambda$$

or, using  $p_c(S_w)$ :

$$K_w(S) = K_w(S_w = 1) \cdot \left( \frac{p_b}{p_c} \right)^\eta \quad \text{for } p_c \geq p_b$$

with  $\eta = 2 + 3\lambda$ . Both models are connected with the water retention curves via the parameters. Similar relations are proposed by van Genuchten (1980).

Hysteresis is present also in the relation  $K_w(S_w)$ . However, it was often considered as weak (Mualem, 1976, Stauffer, 1998). Fig. 3 shows the boundary drying and wetting curves for a sand packing. Consequently hysteresis is often neglected in models. However, clear hysteresis behaviour is present in the relation  $K_w(p_c)$ .

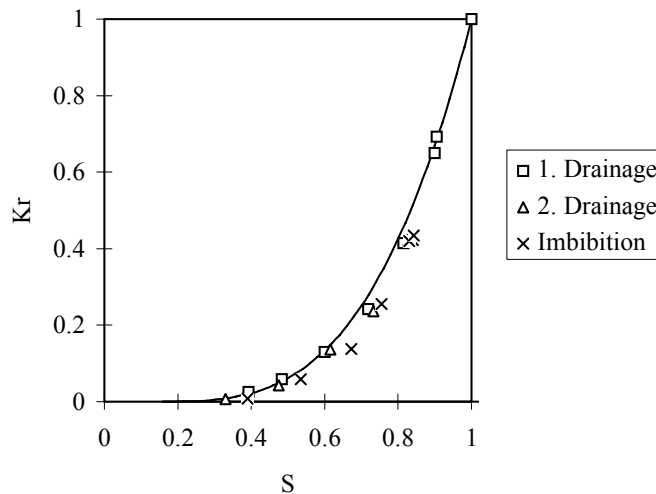


Fig. 3 Boundary drying and wetting curves of the hydraulic conductivity characteristics for a sand packing: Relative hydraulic conductivity  $K_r(S) = K_w(S_w)/K_w(S_w=1)$  as a function of the water saturation  $S = S_w$  (Stauffer, 1997)

## 2. Multiphase flow

**Multiphase flow** concerns the flow of two or more immiscible fluid phases in a porous medium. Therefore several fluids are present simultaneously in the porous medium. An example is given by the system water-air-mineral oil.

## 2.1 Wetting hierarchy

The wetting property of solids by fluids is of paramount importance. This leads to a **wetting hierarchy**, which describes the behavior of two fluid phases towards a solid. (Fig. 4).

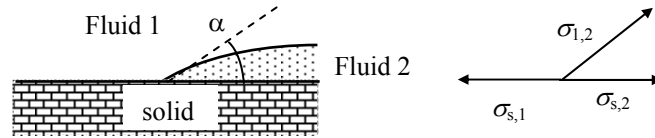


Fig. 4 Two fluids in contact with solid surface.

Mechanical equilibrium is given by (Young's law):

$$\sigma_{1,2} \cos \alpha = \sigma_{s,1} - \sigma_{s,2}$$

The parameter  $\sigma_{1,2}$  is the interfacial tension between fluid 1 and fluid 2, and  $\sigma_{s,1}$  is the interfacial tension between the solid and fluid 1.  $\alpha$  is the wetting angle. No equilibrium is obtained if  $\sigma_{1,2} < \sigma_{s,1} - \sigma_{s,2}$ . Consequently fluid 1 is displaced from the solid surface by fluid 2. Fluid 2 is the wetting fluid and fluid 1 the non-wetting fluid. For quartz or glass surface water is the wetting phase and air the non-wetting one. The **wetting angle** may exhibit a hysteresis effect. Moreover, it is sensitive for impurities on the solid surface.

The **wetting hierarchy** is present also if several fluid phases exist:

- One fluid phase is wetting.
- One fluid phase is non-wetting.
- The remaining fluid phases are wetting-non-wetting, depending on the neighbouring phase (ambivalent behaviour).

For a solid surface, which consists of mineral substance (e.g., quartz sand), water is normally the wetting phase in the presence of air and mineral oil, air is normally the non-wetting phase. Mineral oil is normally the wetting phase in the presence of air alone, and the non-wetting phase in the presence of water alone (Fig. 5a, b).

For a solid surface, which consists of organic substance (e.g., organic soil), mineral oil can be the wetting phase, and air the non-wetting phase in the presence of water and air. Water is then the wetting phase in the presence of air alone, and the non-wetting phase in the presence of mineral oil alone (Fig. 5c).

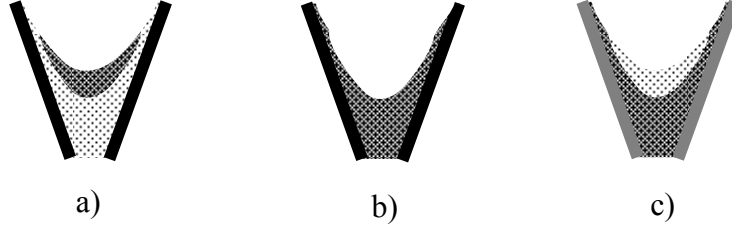


Fig. 5 Schematic wetting behaviour of water (bright) and organic fluid (dark) for mineral solid (a, b, black) and organic solid (c, grey).

A fluid phase gets **immobile**, if the fluid saturation is smaller than the residual saturation  $S_r$  (Fig. 6). Typically, the residual saturation is around 0.1-0.2 for a sand packing. However, if more than two fluid phases are present, the residual saturation of the non-wetting fluid can be very small (Dracos, 1987). Therefore, the residual saturation within a mineral soil can be very small in the presence of water and air (Fig. 6b).



Fig. 6 Residual fluid content: Wetting (black) and non-wetting fluid (white) in a porous medium consisting of mineral solid (grey); a) wetting fluid residual content; b) non-wetting fluid residual content (schematic).

## 2.2 Multiphase flow equation

The **fluid flux equation** for fluid  $f$  is in analogy to Darcy's law:

$$\mathbf{v}_f = -\frac{\mathbf{k}_f(S_f)}{\mu_f} \cdot (\nabla p_f + \rho_f \mathbf{g})$$

The variable  $\mathbf{v}_f$  is the specific flux vector,  $S_f$  is the fluid saturation,  $\mathbf{k}_f$  is the permeability tensor of the porous medium for the fluid,  $\mu_f$  is the dynamic viscosity of the fluid,  $p_f$  is the fluid pressure,  $\rho_f$  is the fluid density. The permeability  $\mathbf{k}_f$  is often connected with the relative permeability  $k_{rf}(S_f)$  and the permeability of the porous medium, with  $\mathbf{k}_f(S_f) = \mathbf{k} k_{rf}(S_f)$ .

The **fluid mass balance** for fluid  $f$  equation states:

$$\frac{\partial(\rho_f n S_f)}{\partial t} = -\nabla \cdot (\mathbf{v}_f) + \rho_f s_f = \nabla \cdot \left( \frac{\mathbf{k}_f(S_f)}{\mu_f} \cdot (\nabla p_f + \rho_f \mathbf{g}) \right) + \rho_f s_f$$

It expresses the rate of change of the fluid mass per unit volume and time of a porous medium. The quantity  $s_f$  is a volumetric fluid source term, which includes in principle also fluid phase change processes.

The **condition for the saturation** of all fluids is:

$$\sum_{f=1}^{n_f} S_f = 1$$

Every fluid mass balance equation contains the variables  $S_f$  und  $p_f$ . However, only  $n_f$  balance equations exist plus the saturation condition. The remaining conditions are obtained by the physical **relationship between two pairs of fluids** (retention curves).

### 2.3 Fluid retention curves

According to the capillary model the capillary pressure is given by the mechanical equilibrium for static conditions:

$$p_{c,1,2} = p_1 - p_2 = \frac{2\sigma_{1,2}}{r}$$

The coefficient  $\sigma_{1,2}$  is the interfacial tension between fluid 1 (wetting fluid) and fluid 2 (non-wetting fluid). The relation between capillary pressure and fluid saturation is often stated as  $p_c(S_{\text{wet}})$  or  $p_c(S_{\text{non-wet}})$ . The saturation  $S_{\text{wet}}$  is the sum of the saturations of all wetting fluid phases including fluid 1. Correspondingly  $S_{\text{non-wet}}$  is the sum of the saturations of all non-wetting fluid phases including fluid 2. For a system with water as the wetting phase (w), mineral oil as wetting-non-wetting phase (h), and air as non-wetting phase (a) it is required that the relations between each of two fluids,  $p_{c,w,h}(S_w)$ ,  $p_{c,w,a}(S_w)$  und  $p_{c,h,a}(S_w+S_h)$  anzugeben.

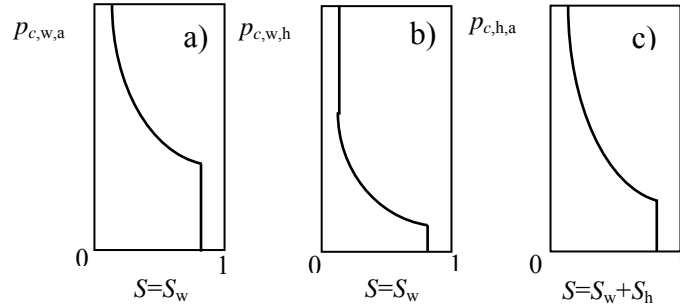


Fig. 7 Relation between capillary pressure and fluid saturation for the system a) water (w)-air (a); b) water (w)-mineral oil (h); and c) mineral oil (h)-air (a).

In principle the relations  $p_c(S)$  have to be determined experimentally. If one relationship exists, e.g., the water retention curve  $S_w(p_{c,w,a})$ , the remaining relations can be obtained approximately with the help of equation 2 (Parker u. a., 1987).

## 2.4 Static fluid distribution

### 2.4.1 Static fluid distribution of a light fluid in a porous medium

The mass  $M_h$  of a light fluid (mineral oil) of density  $\rho_h$  with  $\rho_a < \rho_h < \rho_w$  is injected close to the groundwater table (Fig. 8). Distributed on the area  $A$  yields a total fluid thickness of  $h_t = M_h / (\rho_h A)$ . What is the vertical fluid distribution?

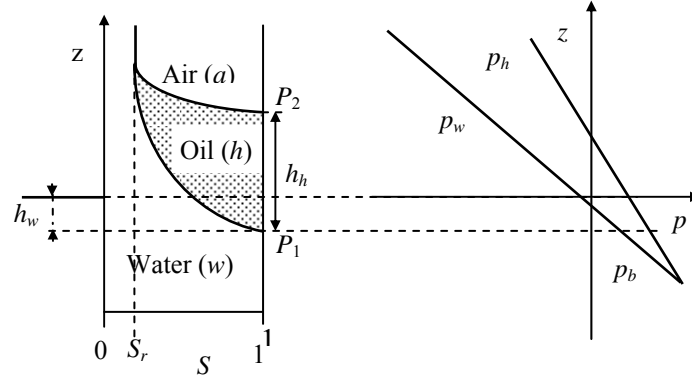


Fig. 8 Static distribution of a light fluid (e.g. mineral oil) in the vicinity of a groundwater table.

Along the water-oil curve the relation  $p_{c,w,h}(S)$  describes a drainage process, in which the wetting fluid is displaced. Assuming a boundary drainage curve described by Brooks-Corey's relationship the water saturation is:

$$S_w = S_{w,r} + (1 - S_{w,r}) \cdot \left( \frac{p_{b,w,h}}{p_{c,w,h}} \right)^\lambda$$

The capillary pressure  $p_{c,w,h}$  equals  $p_h - p_w$ . Due to the static conditions it can be written as:  $p_c = C_1 + (\rho_w - \rho_h)gz$ , where  $C_1$  is a constant. For  $z = -h_w$  the relation holds:  $p_{c,w,h} = -p_{b,w,h}$ . Therefore the constant  $C_1$  can be determined and the water saturation profile can be expressed:

$$S_w = S_r + (1 - S_r) \cdot \left( \frac{p_{b,w,h}}{p_{b,w,h} + (\rho_w - \rho_h)g \cdot (h_w + z)} \right)^\lambda$$

With the help of equation 2 and assuming a wetting angle of zero  $p_{c,w,h}$  is:

$$p_{b,w,h} = p_{b,w,a} \frac{\sigma_{w,h}}{\sigma_{w,a}}$$

The oil pressure in point  $P_1$  is:

$$p_h(z = -h_w) = p_{b,w,h} + \rho_w g h_w$$

Along the oil-air curve the relation  $p_{c,h,a}(S)$  for a wetting process is, again using Brooks-Corey's relation:

$$S = S_r + (1 - S_r) \cdot \left( \frac{p_{b,h,a}}{p_{c,h,a}} \right)^\lambda$$

Since the air pressure is zero  $p_a=0$  the capillary pressure  $p_{c,h,a}$  is equal to  $p_h=C_2+\rho_h g z$  with the constant  $C_2$ . For  $z=h_h-h_w$  the capillary pressure gets  $p_{c,h,a}=p_{b,h,a}$ . Therefore,  $C_2$  can be evaluated and the profile  $S_{w+h}(z)$  can be expressed:

$$S_{w+h} = S_r + (1 - S_r) \cdot \left( \frac{p_{b,w,h}}{p_{b,w,h} + \rho_a g \cdot (z - h_h + h_w)} \right)^\lambda$$

Accordingly,  $p_{b,h,a}$  is:

$$p_{b,h,a} = p_{b,w,a} \frac{\sigma_{h,a}}{\sigma_{w,a}}$$

The oil pressure in point  $P_2$  is:

$$p_h(z_2 = h_h - h_w) = -p_{b,h,a}$$

The oil pressure between point 1 and point 2 can be expressed as follows:

$$p_h(z) = -p_{b,h,a} + \rho_h g \cdot (z_2 - z); \quad z_1 \leq z \leq z_2$$

From the **oil pressure distribution** it can be concluded that negative oil pressure is present in the upper portion of the profile. However in the lower portion the pressure may be positive. Oil can escape into a monitoring well only if it exhibits positive pressure. Therefore, the absence of oil in a monitoring well does not necessarily mean that no oil is present in the aquifer.

The oil pressure in point 1 is:

$$p_h(z_1) = -p_{b,h,a} + \rho_h g \cdot h_h$$

With this the height  $h_w$  can be determined:

$$h_w = \frac{-p_{b,h,a} - p_{b,w,h} + \rho_h g h_h}{\rho_w g}$$

The total amount of oil within the vertical profile can be obtained by integrating the curves  $S_{w,h}(z)$  and  $S_{h,a}(z)$  and using the above mentioned relationships (for  $\lambda \neq 1$ ):

$$h_t = n \cdot (1 - S_r) \cdot \left[ h_h + \frac{1}{\lambda - 1} \left( \frac{p_{b,h,a}}{\rho_h g} - \frac{p_{b,w,h}}{(\rho_w - \rho_h) g} \right) \right]$$

If the total oil thickness  $h_t$  is given, the quantities  $h_h$  and  $h_w$  can be determined directly.

## 2.4.2 Static fluid distribution of a heavy fluid in a porous medium

The mass  $M_h$  of a heavy fluid (e.g., a chlorinated hydrocarbon) of density  $\rho_h$  with  $\rho_a < \rho_w < \rho_h$  is injected at the aquifer bottom (Fig. 9). Distributed on the area  $A$  yields a total fluid thickness of  $h_t = M_h / (\rho_h A)$ . What is the vertical fluid distribution?

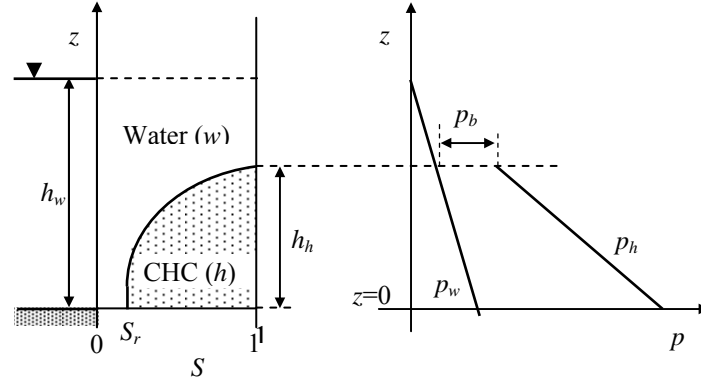


Fig. 9 Static distribution of a heavy fluid (e.g., chlorinated hydrocarbon (CHC)) at the aquifer bottom.

Along the water-CHC curve the relation  $p_{c,w,h}(S)$  applies for a wetting process. According to Brooks-Corey the water saturation is:

$$S = S_r + (1 - S_r) \cdot \left( \frac{P_{b,w,h}}{P_{c,w,h}} \right)^\lambda$$

The capillary pressure  $p_{c,w,h}$  is  $p_h - p_w$ . Due to static conditions it can be stated as  $p_c = C + (\rho_w - \rho_h)gz$  With a constant  $C$ . For  $z = h_h$  it is  $p_c = p_{b,w,h}$ . With this the constant can be evaluated and the profile  $S_w(z)$  can be expressed:

$$S_w(z) = S_r + (1 - S_r) \cdot \left( \frac{P_{b,w,h}}{P_{b,w,h} + (\rho_h - \rho_w)g \cdot (h_h - z)} \right)^\lambda$$

For the CHC the profile is  $S_h(z) = 1 - S_w(z)$ . With the help of equation 2 and assuming a wetting angle of zero the quantity  $p_{c,w,h}$  gets:

$$P_{c,w,h} = P_{b,w,a} \frac{\sigma_{w,h}}{\sigma_{w,a}}$$

the total CHC thickness  $h_t$  within the vertical profile can be obtained by integrating the vertical fluid profile using the above relations (for  $\lambda \neq 1$ ):

$$h_t = n \cdot (1 - S_r) \cdot \left[ h_h - \frac{1}{\lambda - 1} \left( -\frac{p_{b,w,h}}{(\rho_w - \rho_h)g} + \left( \frac{p_{b,w,h}}{p_{b,w,h} + h_k \cdot (\rho_w - \rho_h)g} \right)^\lambda \right) \right]$$

If the total CHC thickness is given, the quantity  $h_h$  can be obtained directly.

## 2.5. Crude estimation of mineral oil infiltration and migration

The following method describes a **crude estimation of the infiltration of mineral oil** due to an oil spill and the subsequent **migration in an unconfined aquifer**. Consider the following simplified situation. The amount  $V_h$  of mineral oil is spilled on the soil surface. Subsequently a pond is forming. Assume that the pond is circular with radius  $R_h$  and has constant thickness  $h_s$ , and is forming immediately. The oil is starting to **infiltrate** into the soil (zone 1, Fig. 10). For the infiltration it is assumed that the oil is in this stage only vertically displaced in a homogeneous soil. The oil saturation  $S_{h,1}$  corresponds to a quasi-saturation. The capillary fringe above the water table is located in a depth  $H_1$ . The thickness of the capillary fringe is approximated by the mean capillary pressure head  $h_{c,w,a}$ .

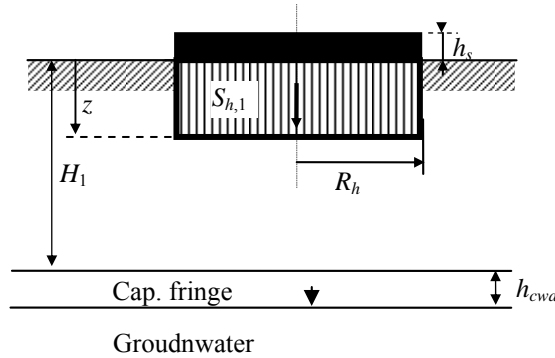


Abb. 10 Crude assessment of oil infiltration: Phase I.

Fed by the oil pond the oil phase in the soil is propagating vertically down to the depth  $z_1$ : according to a simple volume balance:

$$z_1 = \frac{V_h}{\pi R_h^2 n S_{h,1}}$$

the parameter  $n$  is the soil porosity. The **velocity of the oil front** can be formulated as:

$$u = \frac{dz}{dt} = \frac{K_h(S_{h,1})}{n S_{h,1}} \cdot \left( 1 + \frac{(h_s + h_{c,h,a})}{z} \right)$$

where the penetration depth is  $z \leq z_1$ . The variable  $h_s(t)$  is the transient thickness the oil pond, and  $t$  is the time. The parameter  $K_h(S_{h,1})$  is the **oil conductivity** for the oil saturation  $S_{h,1}$ . The

latter can be calculated from the hydraulic conductivity for saturation  $K_w(S_w=1)$  using the corresponding kinematic viscosities  $\nu_w$  and  $\nu_h$  according to:

$$K_h(S_{h,1}) = \frac{K_w(S_w=1)\nu_w}{\nu_h} K_{r,h}(S_{h,1})$$

the relative conductivity  $K_{r,h}$  for oil can be assessed by using Brooks und Corey's (1966) relation. The exponent  $\varepsilon$  is frequently between 3 and 4. The **time span**  $t_1$  until the soil depth  $z_1$  is reached can be obtained by integration of the velocity equation:

$$t_1 = \frac{n S_{h,1}}{K_o(S_{h,1})} \left[ z_1 - \left( h_{s,\max} / 2 + {}_w h_{c,h,a} \right) \ln \left( \frac{z_1}{h_{s,\max} / 2 + {}_w h_{c,h,a}} + 1 \right) \right]$$

After the oil pond runs dry the quasi-saturated oil body separates from the soil surface and migrates downwards and leaves behind a zone with **residual oil saturation**  $S_{r,h,1}$ . If the location of the capillary fringe is very deep the oil will be completely trapped in the soil zone with residual oil saturation. The corresponding depth will be (Fig. 12):

$$z_1 = \frac{h_s}{n S_{r,h,1}}$$

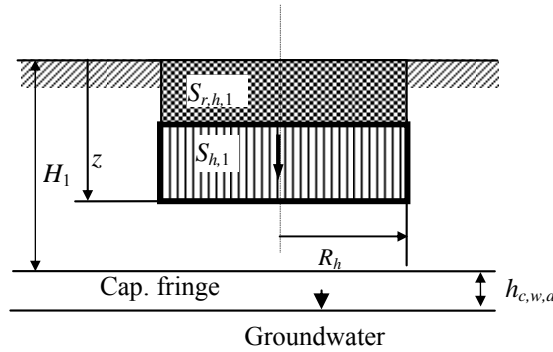


Abb. 11 Crude assessment of oil infiltration: Phase II. (vertical hatching: mobile oil; dark hatching: immobile (residual) oil).

If the capillary fringe is close to the soil surface the corresponding depth is denoted as  $z_1'=H_1$ . assuming a oil head gradient of one the **oil front reaches the capillary fringe** at time  $t$ . The corresponding quasi-saturated oil thickness can be assessed by a volume consideration:

$$V_h = H \pi R_h^2 n S_{h,1} + (H_1 - H) \pi R_h^2 n S_{r,h,1}$$

$$H = \frac{V_h}{\pi n (S_{h,1} - S_{r,h,1}) R_h^2} - H_1 \frac{S_{r,h,1}}{S_{h,1} - S_{r,h,1}}$$

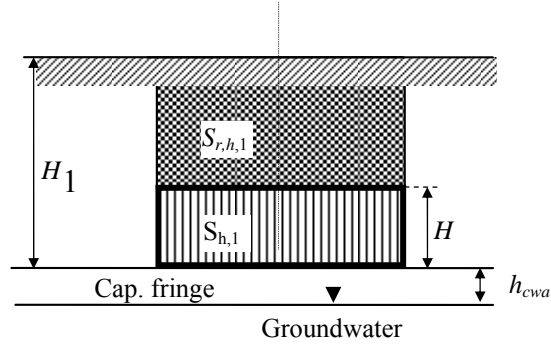


Fig. 12 Crude assessment of oil infiltration: Phase III.

Provided a static oil pressure within the oil phase is built up, the oil body ‘sinks’ (Fig. 13). The **maximum penetration depth** can be assessed by using static considerations (swim condition, zone 3):

$$(z_{\max} - h_{c,w,a}) \cdot \rho_w = (z_{\max} + H_D - d h_{c,h,a}) \rho_h$$

The quantity  $H_D$  is the remaining oil thickness above the capillary fringe. It can be determined by the volume balance:

$$H S_{h,1} = H_D S_{h,1} + z_{\max} S_{h,3} + (H - H_D) S_{r,h,1}$$

$$H_D = H - z_{\max} \frac{S_{h,3}}{S_{h,1} - S_{r,h,1}}$$

Therefore the maximum penetration depth below the capillary fringe can be determined:

$$z_{\max} = \frac{H \rho_h - d h_{c,h,a} \rho_h + h_{c,w,a} \rho_w}{\rho_w - \rho_h + \rho_h \frac{S_{h,3}}{S_{h,1} - S_{r,h,1}}}$$

where  $d h_{c,w,a}$  is the mean capillary pressure head of oil-air for a drainage process,  $S_{h,3}$  is the oil saturation within zone 3,  $\rho_w$  is the water density, and  $\rho_h$  is the oil density.

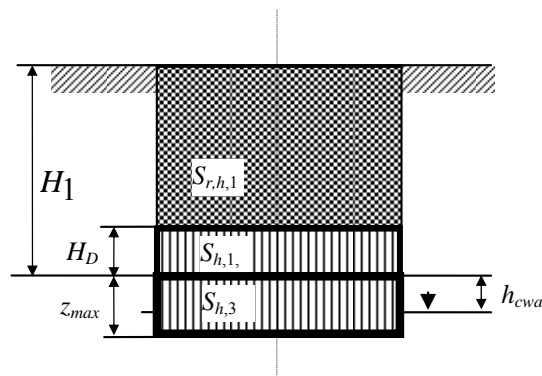


Fig. 13 Crude assessment of oil infiltration: Phase IV.

It is assumed that a horizontal oil migration starts after sinking of the oil body in a sequential manner. Further it is assumed that the movement of the oil is **radial**, and that the ‘quasi’-horizontal migration occurs in a oil layer of thickness  $w h_{c,h,a}$  (corresponding to the mean capillary pressure head oil-air for wetting, Zone 2). The radial migration stops when the oil layer thickness below the infiltration domain reaches the value  $d h_{c,w,a}$  (corresponds to the mean capillary pressure head for oil-air at drainage). A volume consideration yields:

$$H R_h^2 S_{h,1} = d h_{c,h,a} R_h^2 S_{h,1} + z_{max} R_h^2 S_{r,h,3} + (R_{max}^2 - R_h^2) w h_{c,h,a} S_{h,2}$$

where  $S_{h,2}$  is the oil saturation in zone 2.

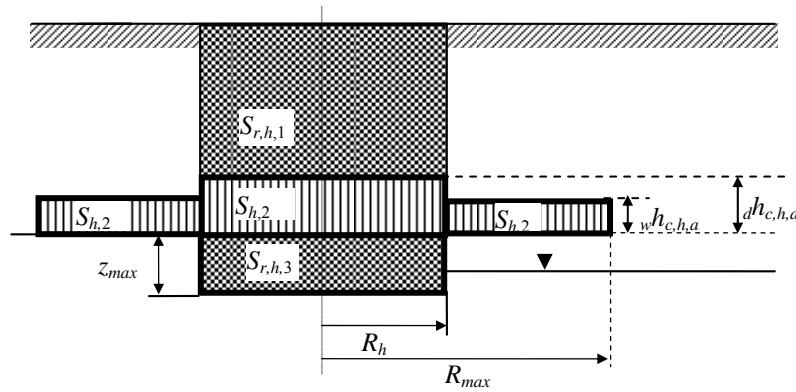


Fig. 14 Crude assessment of oil infiltration: Phase V.

This leads to a maximum radius  $R_{max}$  of mobile oil:

$$R_{max}^2 = R_0^2 \cdot \left( 1 + \frac{H S_{o1} - d h_{coa} S_{o1} - z_{max} S_{ro3}}{w h_{coa} S_{ro2}} \right)$$

For the **last migration phase** it is assumed that the propagation of mobile oil occurs in the capillary fringe **in the flow direction of the groundwater**. The propagation stops when all oil at residual saturation and therefore immobile. (Fig. 15). However this process is very slow. Moreover, it will be strongly affected by a vertical movement of the groundwater table, which leads to a vertical smearing of the oil and which is difficult to assess. Therefore the maximum horizontal migration distance is smaller. Nevertheless the theoretical maximum migration length can be determined using volume considerations.

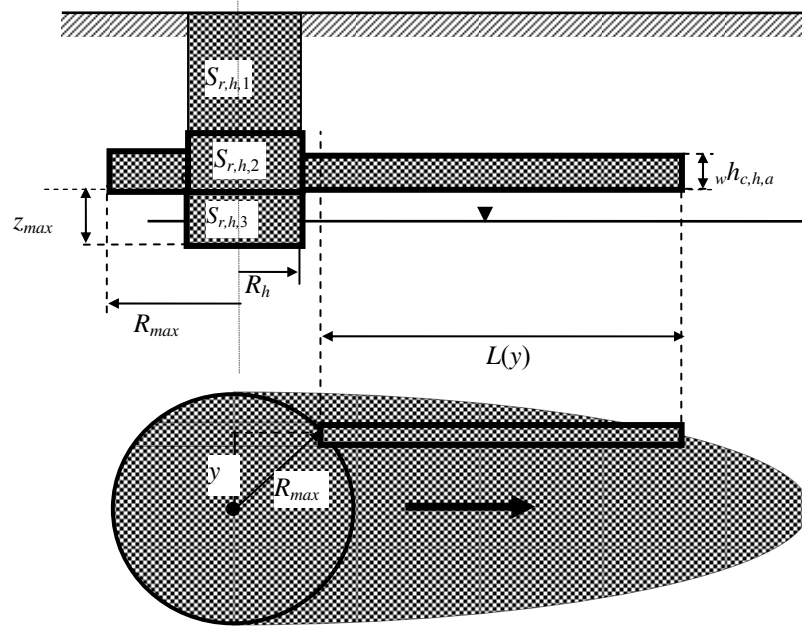


Fig. 15 Crude assessment of oil infiltration: Theoretical end situation.

The volume consideration for  $y \leq R_h$  yields:

$$L(y) w h_{c,h,a} S_{r,h,2} = 2 \cdot (S_{h,2} - S_{r,h,2}) \cdot \left( w h_{c,h,a} \sqrt{R_{\max}^2 - y^2} + d h_{c,h,a} \sqrt{R_h^2 - y^2} \right)$$

The length  $L(y)$  is the length of a strip with unit width measured from the centre of the infiltration cylinder in the  $y$ -direction normal to the groundwater flow direction. The length  $L(y)$  is:

$$L(y) = \frac{2 \cdot (S_{h,2} - S_{r,h,2}) \cdot \left( w h_{c,h,a} \sqrt{R_{\max}^2 - y^2} + d h_{c,h,a} \sqrt{R_h^2 - y^2} \right)}{w h_{c,h,a} S_{r,h,2}}$$

and for  $R_0 < y \leq R_{\max}$ :

$$L(y) = \frac{2 \cdot (S_{o2} - S_{ro2}) \cdot \sqrt{R_{\max}^2 - y^2}}{S_{ro2}}$$

For  $y=0$  the **maximum oil horizontal displacement in the direction of the groundwater flow** is obtained. It's recalled that this measure is an **upper limit** because oil is also vertically smeared due to vertical movement of the groundwater table and therefore the maximum displacement is smaller.

## Literature

Brooks R. H., and A. T. Corey, Properties of porous media affecting fluid flow. J. Irrig. Drain. Div. ASCE, 92 (2) 61-88, 1966.

- Corey A. T., Mechanics of heterogeneous fluids in porous media. Water Resour. Publ. Fort Collins, Colorado, 1977.
- Dracos T., Multiphase flow in porous media, In: Modelling and applications of transport phenomena in porous media (Eds. J. Bear, and J.-M. Buchlin, Kluwer, Dordrecht, p. 195-220, 1987
- Mualem Y., A new model for predicting the hydraulic conductivity of porous media. Water Resour. Res. 12 (3) 513-522, 1976.
- Parker J. C., R. J. Lenhard, and T. Koppusami, A parametric model for constitutive properties governing multiphase flow in porous media. Water Resour. Res. 23 (4) 618-624, 1987.
- Stauffer F. and T. Dracos, Experimental and numerical study of water and solute infiltration in layered porous media. J. Hydrology, 84, 9-34, 1984.
- Stauffer F., Strömungsprozesse im Grundwasser, Konzepte und Modelle. vdf, Hochschul-Verlag an der ETH Zürich, 1998.
- Stauffer, F., H.-J. Franke, and T. Dracos, Hysteretic storativity concept for aquifer simulation. Water Resour. Res. 29 (9) 2307-2314, 1992.
- van Genuchten R., A closed form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 44, 892-898, 1980.